The Disintegration (or not) of the Nonlinear Internal Tide

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LONG-TERM GOALS

This research is aimed at studying the underlying dynamics of, and identifying the conditions that control, the disintegration of the internal tide into large-amplitude internal solitary-like waves with emphasis on the tides and waves in the South China Sea.

OBJECTIVES

The objectives are to use a combination of theoretical and numerical models to study the evolution of the internal tide and its possible disintegration into internal solitary waves. A central aspect of this work is to explore the role of rotation in the process. Rotation permits the presence of periodic, nonlinear inertia-gravity waves (i.e., the tide) that can act as attractors and arrest the steepening of the internal tide, and hence affect the production of the shorter solitary-like waves (Gerkema, 1996). In light of recent observations of strongly nonlinear internal solitary waves in the South China Sea (e.g. Duda *et al*, 2004; Ramp *et al*., 2004; Zhao and Alford, 2006) and numerous other locations, an important objective is to allow for fully nonlinear waves. A further objective is to test these theories and models with observations obtained from the NLIWI South China Sea DRI in order to improve the ability to predict the arrival of large-amplitude internal solitary waves.

APPROACH

The approach combines theoretical wave evolution models and numerical solutions of these models and solutions of the full Navier-Stokes equations. The theoretical models require some simplifications including restriction to two-layer flows and one-dimensional propagation. The presence of rotation requires flow in the direction transverse to the propagation; however, variations of properties in this direction are ignored. The theory is an extension of the fully-nonlinear, weakly non-hydrostatic internal wave theory of Miyata (1988) and Choi and Camassa (1999) to include rotation. In most cases these model equations are solved numerically using modern, high-order schemes. This theory is complemented using a 2.5-dimensional Navier-Stokes numerical model that permits continuous stratification and eliminates restrictions associated with the long-wave assumption in the theory. Variable topography has been included in both the theory and the numerical models.

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14. ABSTRACT
This research is aimed at studying the underlying dynamics of, and identifying the conditions that control, the disintegration of the internal tide into large-amplitude internal solitary-like waves with emphasis on the tides and waves in the South China Sea.

15. SUBJECT TERMS

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The theoretical and modeling results are being compared against PIES observations by David Farmer (URI) and other NILIWI investigators of the low-mode internal tide evolution across the South China Sea from just west of Luzon Strait to the Chinese shelf.

Laboratory experiments at the LEGI-Coriolis facility will be conducted September 28-October 16, 2009 to test the numerical and theoretical work on the effects of rotation on internal solitary waves (see WORK COMPLETED and Helfrich (2007) and Grimshaw and Helfrich (2008))

WORK COMPLETED

The Miyata/Choi-Camassa (MCC) theory has been extended to include rotation (2.5 dimension MCC-f model) and a numerical scheme has been developed and tested. In Helfrich (2007) the effects of weak background rotation on the evolution of large-amplitude internal solitary waves has been studied. This work was continued with Roger Grimshaw at Loughborough University in a study of the nonlinear wave packet dynamics (Grimshaw and Helfrich, 2008).

A second completed part of this project is a study of the nonlinear internal tide solutions of the two-layer MCC-f equations (in the hydrostatic limit) and the disintegration of an initially sinusoidal internal tide into remnant internal tide and high-frequency solitary-like waves (Helfrich and Grimshaw, 2007). In Helfrich (2008) the hydrostatic, fully-nonlinear nonlinear internal tide theory is extended to include continuous stratification and conditions of the South China Sea are briefly explored.

Two related projects on internal wave generation have been completed. A theory for steady gravity currents in continuously stratified fluids is developed (White and Helfrich, 2008). The theory is used to determine the conditions when unsteady nonlinear internal waves are generated ahead of the gravity current. Numerical solutions (2-dimensional Navier-Stokes equations) of the initial value dam-break problem are used to test the theoretical results. Da Silva and Helfrich (2008) examined the generation of nonlinear internal waves by tidal flow though a channel. From satellite remote sensing imagery and some in situ current data from Race Point Channel (Cape Cod), it is found that two packets can be generated on a single ebb tide due to a novel double passage through resonance mechanism.

A theoretical model for large-amplitude solitary waves with trapped cores has been developed (Helfrich and White). The model avoids the limitations to either weakly nonlinear waves or unphysical density distributions in the trapped cores of earlier theories. Companion 2-dimensional Navier-Stokes numerical solutions support the model results.

RESULTS

Earlier work on the role of rotation on the evolution of an internal tide had been restricted to weakly nonlinear waves (e.g., Gerkema, 1996; Holloway *et al.*, 1999). Gerkema (1996) found that with rotation the fission of the tide into solitary wave packets could be inhibited. This was due to the presence of rotational dispersion, which could balance the nonlinearity to give hydrostatic nonlinear tide (inertia-gravity) solutions that prevented further steepening of the tide, and thus the emergence of shorter solitary waves. However, observations such as those in the SCS frequently show both tides and waves with amplitudes beyond the restrictions of weakly nonlinear theory, the disintegration of an initial long internal tide has been studied using a fully-nonlinear two-layer theory (termed MCC-f) (Helfrich and
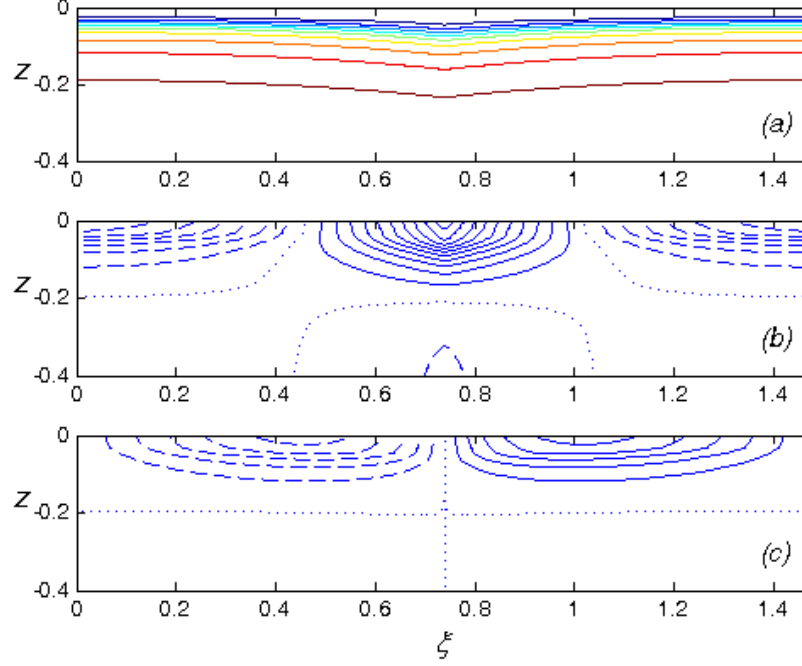
Figure 1. Fully nonlinear, hydrostatic tide solution for the diurnal tide in the South China Sea. The density field is shown in (a), the x- velocity field in (b), and the transverse, y- velocityin (c). Here ξ = x-ct, where c is the wave phase speed. In (a) and (b) the solid (dashed) contours indicate positive (negative) velocities.This solution is the maximum allowable amplitude and in dimensional variables the wave is 363 km and the maximum isopycnal displacement is 93 m.

Grimshaw, 2007) and in continuously stratified conditions (Helfrich, 2008). The essential dynamical feature regardless of the stratification is that in the hydrostatic limit, there exist exact, periodic, nonlinear inertia-gravity solutions. These are low-mode nonlinear tides. These tides are robust to weak nonhydrostatic effects. Numerical solutions of the two-layer model show that the disintegration of an initial sinusoidal linear internal tide is closely linked to the presence of these nonlinear waves. The initial tide steepens due to nonlinearity and sheds energy into short solitary-like waves. The disintegration is halted as the long wave part of the solution settles onto one of the nonlinear hydrostatic solutions, with the short solitary waves superimposed. The degree of disintegration is a function of initial amplitude of the tide and the properties of the underlying nonlinear hydrostatic solutions, which, depending on stratification and tidal frequency, exist only for a finite range of amplitudes. There is a lower threshold below which no short solitary waves are produced. However, for initial amplitudes above another threshold, given approximately by the energy of the limiting nonlinear hydrostatic inertia-gravity wave, most of the initial tidal energy goes into solitary waves.

The MCC-f theory is informative; however, in the present form, it is restricted to two-layers. This limitation has been lifted in Helfrich (2008) where the nonlinear tide theory is extended to allow continuous stratification. Figure 1 shows the limiting (maximum amplitude) diurnal internal tide solution computed for the stratification from the northern South China Sea. The tide has a corner shape with maximum isopycnal displacements of 93 m.

These models are being compared to the NLIWI SCS observations. The model offers an explaination for the observations of Zhao and Alford (2006). They showed that the arrival of large nonlinear internal solitary-like waves at the western side of the basin is generally linked to phases when the generating tide in the Luzon Strait is dominantly semi-diurnal. When the Luzon tide is dominantly diurnal, the large solitary waves are absent despite the fact that the Luzon semi-diurnal and diurnal barotropic tides are of similar magnitude. This is in agreement with the continuously stratified model just described. While diurnal nonlinear tide solutions were found for the SCS conditions, it was not possible to find nonlinear tides at the semidiurnal frequency. As a result, an initial sinusoidal internal tide at the
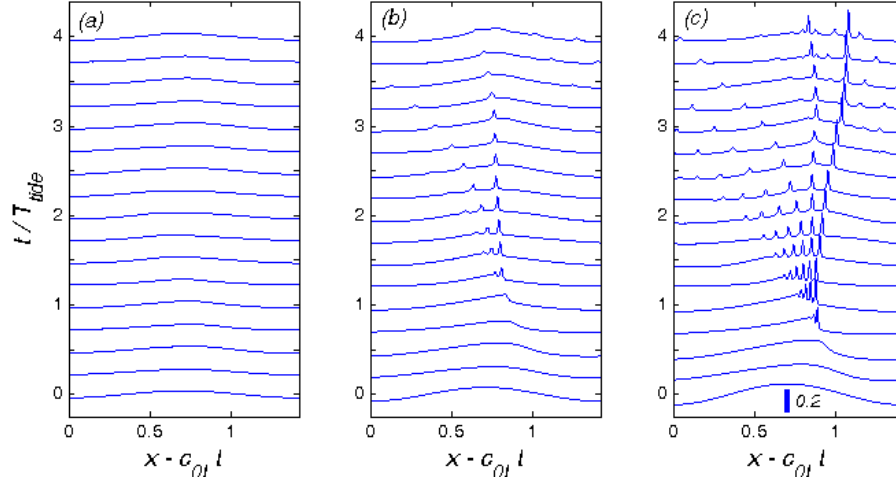

Figure 2. Numerical solutions of a 2.5 dimensional nonhydrostatic numerical model initiated with a linear diurnal internal tide. The stratification used is representative of the SCS. The figure shows the (nondimensional) surface current every quarter period of the linear tide for four tidal periods. The initiall tidal amplitudes are 25m (a), 50m (b), and 75m (c). The production of NLIWs is inhibited at this tidal frequency by the presence of stable nonlinear tide solutions.
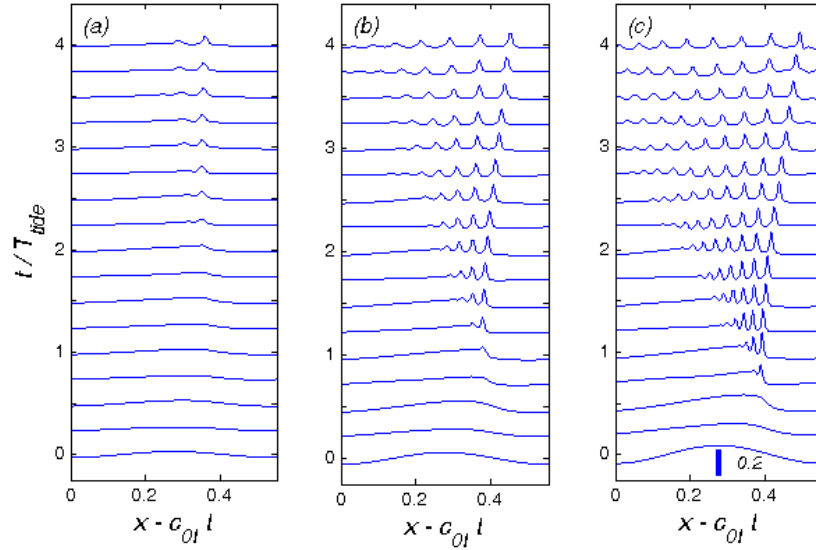

Figure 3. Same as Figure except the initial tide is at the semi-diurnal frequency. In all cases the initial tide disintegrates into NLIWs.

semidiurnal frequency will disintegrate almost completely into short nonlinear solitary-like waves, which the diurnal tide is more likely to remain intact and inhibit the production of NLIWs. This is illustrated by solutions of the 2.5-dimensional nonhydrostatic numerical model initiated with linear internal tides at the diurnal (Figure 2) and semi-diurnal (Figure 3) frequencies for initial tidal.
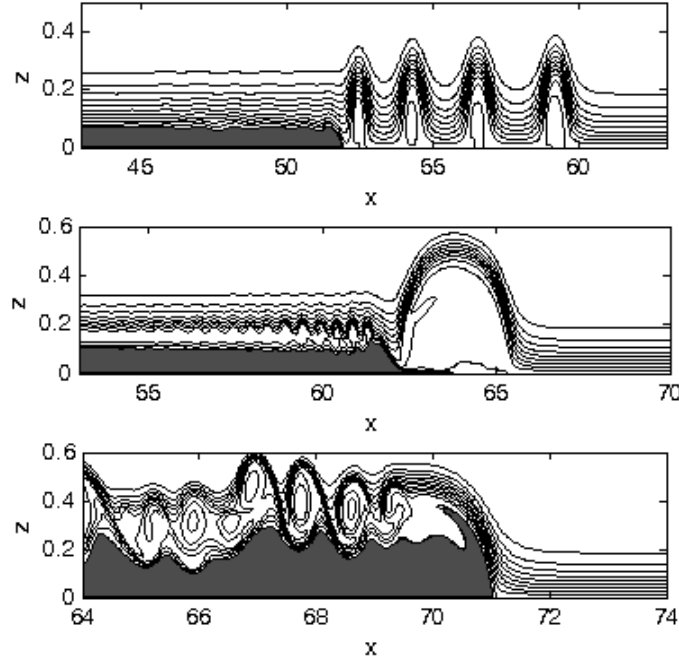

Figure 5. Numerical solutions of the Navier-Stokes equations showing the leading portion of a gravity current (gray shading) and upstream nonlinear internal waves. The gravity current is produced by a dam-break far to the left in each frame. The depth of the dammed fluid increases from the TOP to the BOTTOM panel. In the TOP panel the gravity current generates an upstream group of nonlinear internal waves that outrun the gravity current. The MIDDLE panel shows a case where the gravity current generates one very large wave that is phase locked to the front of the gravity current. In the BOTTOM panel no upstream waves are produced and the gravity current properties (speed, height and ambient density and velocity fields) are in good agreement with the theoretically predicted dissipationless (conjugate state) gravity current.

amplitudes of 25, 50 and 75 m. The semi-diurnal tides always disintegrate into shorter NLIWs, while the production of NLIWs is inhibited at the diurnal frequency

Continuing work is focused on direct comparisons of these models with SCS observations such as the PIES moorings of David Farmer (URI) and includes the effects of variable topography and mixed tides.

Recent observations of NLIW generation by the Columbia River plume (Nash and Moum, 2005) indicated that large NLIWs can be generated by the propagation of a gravity current into a stratified fluid. They suggested that the waves are released from the gravity currents as the gravity current speed passes from super- to sub-critical (compared to the linear long wave phase speed). A theory for gravity currents propagating into a stratified fluid with a general density profile has been derived (White and

Helfrich, 2008). The theory predicts steady gravity currents with constant front speed over a range of current thicknesses and speeds. The solutions have two limits: an energy conserving solution (the conjugate state, or limiting amplitude solitary wave with a trapped core) and a condition of incipient convective instability. Nonhydrostatic model simulations of a lock release initial value problem show that internal waves are generated and propagate ahead of the gravity (see Figure 5). The conditions leasing to internal wave generation correspond to the range in which the steady gravity current theory fails (i.e., gravity currents with speeds below the convective overturning limit). This is generally not the critical Froude number based on the ambient long wave phase speed and, in contrast to Nash and Moum (2005), wave generation does not depend on deceleration of the gravity current from super- to
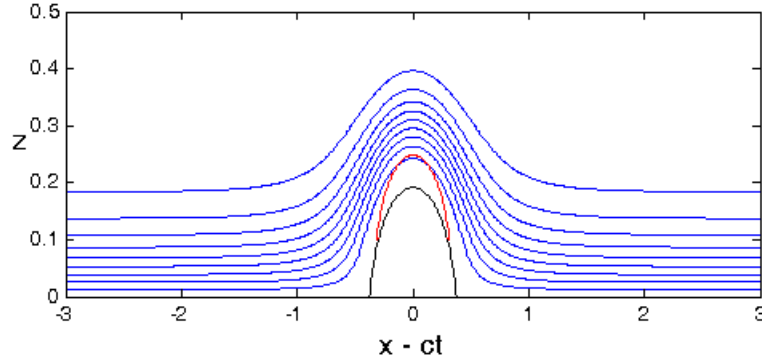

Figure 6. A theoretical solution for an internal solitary wave of elevation with a trapped core. The boundary of the trapped core is shown by the black line. The blue lines show isopycnals and the red line bounds the region above the core of Richardson number less than 0.25. The wave is propagating to the right with nondimensional phase speed 0.41. The stratification is given by a hyperbolic tangent and the core has the density of the ambient stratification at z = 0. For this stratification the linear long wave phase speed is 0.226 and trapped cores occur for solitary wave phase speeds greater than 0.330. Note that only the lower half of the water column is shown.

sub-critical conditions. The generation is more appropriately viewed as a resonant generation process (c.f. Grimshaw and Smyth, 1986; Melville and Helfrich, 1987).

Generation nonlinear internal waves by tidal flow though a strait is usually ascribed to the radiation of an internal tide (or lee-wave) that subsequently steepens to form a wave packet. One packet is formed on each ebb or flood phase of the tide with the two packets propagating in opposite directions. Recently analysis of Synthetic Aperture Radar images and in situ flow data revealed a novel mechanism in which two packets (propagating in the same direction) may be generated on a single ebb phase of the tide through Race Point Channel in Massachusetts Bay (da Silva and Helfrich, 2008). The images and in situ measurements of the flow in the channel are used to infer the generation mechanism of the waves within the Channel. The generation occurs at the same location within the channel, but at different phases of the ebb tide. The two individual packets of waves result from flow passage through resonance (where the Froude number is one). One packet is generated as the flow passes through the transcritical regime during the acceleration phase of the (ebb) tidal current, and another packet is generated during the deceleration phase. Both packets propagate upstream into Massachusetts Bay when the tide slacks, but with slightly different propagation directions. The conditions leading to the

double generation are common and this mechanism should be expected to occur elsewhere. Current work is directed at a modeling study of the generation process.

A theoretical model for large-amplitude internal solitary waves with trapped cores has been developed. Earlier theoretical work on trapped core waves was either limited to weakly nonlinear waves in nearly uniform stratification (Derzho and Grimshaw, 1997) or was based on solutions of the fully-nonlinear Dubriel-Jacotin-Long (DJL) equation (e.g., Tung *et a*l., 1982; Turkington *et al*., 1991; Brown and Christie, 1998). However, the latter DJL-based models have a significant fault. The trapped cores contain fluid with densities outside the range of the ambient stratification and the density field within the trapped core is statically unstable. Thus these solutions are physically suspect. This situation is rectified in a model with a core of uniform density and velocity equal to the phase speed, but unknown size and shape. The flow outside the core governed by the DJL equation. Figure 6 shows an example
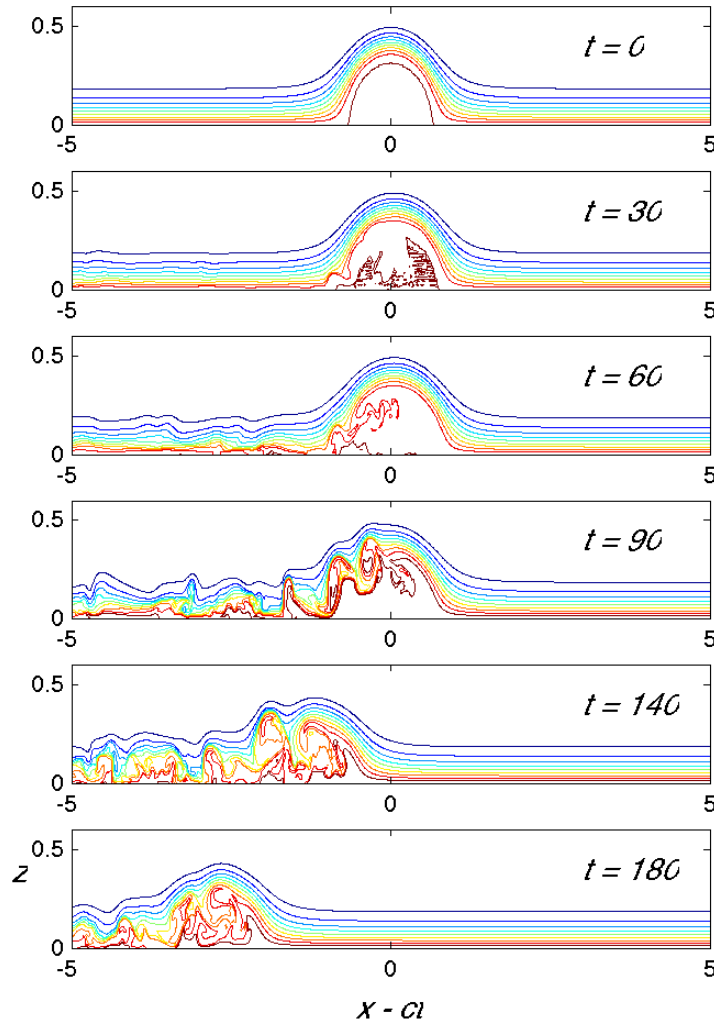

Figure 6. An example of a trapped core solution that is unstable to shear instability. This case has the same ambient stratification as Figure 6 except the initial wave (t = 0) from the theoretical model has c = 0.45 and is larger amplitude. The minimum Richardson number over the core at t = 0 is 0.13. The lower five panels show the evolution of the wave computed with a two-dimensional nonhydrostatic model. Shear instability destroys the initial wave. Ultimately a new, smaller trapped core wave will emerge.

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solution of this modified DJL theory for a large wave with a trapped core. The theoretical solutions may have low Richardson numbers less than 0.25 in the ambient fluid. If the Richardson number is low enough the theoretical solution is susceptible to shear instability that can destroy the initial wave as shown in Figure 7. This work is reported in Helfrich and White (submitted).

IMPACT/APPLICATIONS

The ubiquitous nature of large amplitude internal solitary waves in the world's coastal oceans and marginal seas is clear from observations. These waves are can have significant effects on coastal mixing through breaking as they propagate and shoal, and they may also lead to substantial horizontal mass transport. Since the waves are frequently generated through the radiation of an internal tide by barotropic tidal flow over localized topography (as is apparently the case at in the Luzon Strait), this work will help understand what fraction of the energy put in at the tidal frequency ends up as internal solitary waves, the space and time scales for that transformation, and the characteristics of the resulting solitary-like waves.

The study of the coupling of gravity currents and NLIWs and the novel double generation process observed to occur at Race Point Channel both involve less well understood, but potentially important, NLIW generation mechanisms.

The theoretical model of large-amplitude internal solitary waves eliminates the restriction to weakly nonlinear conditions and removes a serious flaw of earlier solutions of the fully nonlinear DJL equation. The trapped core waves are limited in amplitude by shear instability that can destroy the wave.

This basic theoretical and numerical work will be directly applicable to the analysis and interpretation of the NLIWI South China Sea observations. To this end, current efforts are aimed at direct comparisons of the two-layer MCC-f and continuously stratified models with the PIES observations by David Farmer (URI) of the evolution of the internal tide and NLIWs across the northeastern South China Sea.

RELATED PROJECTS

This work is directly related to the ONR NLIWI South China Sea DRI.

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